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~~B.Sc - part - II~~ Class - I.Sc (XII)

Ex 1.2. Relations and functions

Q.1. Let $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. Show that f is one-one.

Soln: It is given that $A = \{1,2,3\}$ and $B = \{4,5,6,7\}$

Now, $f: A \to B$ is defined as $f = \{(1,4),(2,5),(3,6)\}$.

$\therefore f(1) = 4,\ f(2) = 5,\ f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one.

Q.2. In each of the following cases, state whether the functions is one-one, onto or bijective, Justify answer.

(I) $f: R \to R$ defined by $f(x) = 3-4x$

(II) $f: R \to R$ defined by $f(x) = 1+x^2$.

Soln: It is given that $f: R \to R$ is defined by $f(x) = 3-4x$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow \cancel{3} - 4x_1 = \cancel{3} - 4x_2 \Rightarrow -\cancel{4}x_1 = -\cancel{4}x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

For any real number y in R, there exists $\frac{3-y}{4}$ in R such that $f\left(\frac{3-y}{4}\right) = 3 - \cancel{4}\left(\frac{3-y}{\cancel{4}}\right) = \cancel{3} - \cancel{3} + y = y$

$\therefore$ f is onto. Hence, f is bijective.

II It is given that $f: R \to R$ is defined as $f(x) = 1+x^2$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 1+x_1^2 = 1+x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$.

For instance, $f(1) = 1^2 = 1+1 = 2$ and $f(-1) = 1+(-1)^2 = 1+1 = 2$

$\therefore f(1) = f(-1) = 2$

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Rest part of Q.2.

$\therefore$ $f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$

$\therefore$ f is not one-one.

Let us consider an element -2 in Co-domain R.

It is seen that $f(x) = 1+x^2$ is positive for all $x \in R$.

Thus, there does not exist any x in domain R such that $f(x) = -2$.

$\therefore$ f is not onto. Hence, f is neither one-one nor onto.

Q.3. Let A and B be sets. Show that $f: A\times B \to B\times A$ such that $f(a,b) = (b,a)$ is bijective function.

Soln. It is given that

$f: A \times B \to B\times A$ is defined as

$f(a,b) = (b,a)$

Let $(a_1, b_1), (a_2, b_2) \in A\times B$

such that $f(a_1, b_1) = f(a_2, b_2) \Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2 \Rightarrow (a_1, b_1) = (a_2, b_2)$.

$\therefore$ f is one-one.

Now, let $(b,a) \in B\times A$ of any element

Then, there exists $(a,b) \in A\times B$ such that

$f(a,b) = (b,a)$.

$\therefore$ f is onto.

Hence, f is bijective.